Using scatterplots to teach the critical power concept

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The critical power (CP) concept has received renewed attention and excitement in the academic community (5). The CP concept was originally conceived as a model derived from a series of exhaustive, constant-load, exercise bouts [for a review, see Hill et al. (8)]. All-out exercise testing (2, 15, 18) has made quantification of the parameters for the two-component model easier to arrive at, which may increase use of the CP concept (16). The purpose of this article is to summarize a tutorial aided with scatterplots created using Microsoft Excel that I have found useful for illuminating the CP concept.

CP represents an exercising intensity that can be performed, at least theoretically, without exhaustion occurring and represents the maximal steady state for lactate and O2 uptake (\(\dot{V}O_2\)) (17). Conversely, when intensities exceed CP, expenditure of the finite anaerobic capacity (\(W\)) ensues along with the accumulation of metabolites known to impede skeletal muscle contractions (12). Tightly coupled with the expenditure of \(W\) is the rise in pulmonary \(\dot{V}O_2\) toward maximal \(\dot{V}O_2\) (\(\dot{V}O_{2\text{max}}\)) (9). Indeed, the higher the intensity above CP, the more rapid the expenditure \(W\) and achievement of \(\dot{V}O_{2\text{max}}\) occurs (19).

Application of mathematical models for the CP concept provides a deeper understanding of high-intensity exercise performance (14). At the root of the CP concept is the inverse time model (20), a model that can be nebulous to students and professionals alike. The CP model is equally valid for running (6); however, gravitational force on body mass is dropped from the equations. Thus, the mathematics for applying the CP model are simpler for running versus cycle ergometry. Moreover, the relationship between distance and running time is more intuitive to students and can be used to resolve the parameters of critical speed (CS) and the speed-time curvature constant (\(D'\)).

Creating a Scatterplot of Running Data to Derive CS and \(D'\)

The trendline feature in Excel scatterplots enables students an opportunity to create a linear regression equation with relative ease. The feature allows for a progressive introduction to the mathematics associated with the CP model and serves as a basis for discussing the physiological mechanisms governing the maximal aerobic steady state and the curvature constant for the speed-time or power-time relationship. To begin, have the student enter the data shown in Table 1 and create an x-y scatterplot of the distance data (column A, y-axis) and the time limit (TLIM; column B, x-axis). By right-clicking over the speed-time series of data, the student can select “add trendline” and highlight the option to “Display Equation on Chart.” Figure 1, left, shows the completed graph with axes and units of measurement labeled. The regression equation for solving distance includes the slope (or CS) and the y-intercept (or \(D'\)). The result yields the first two-component model equation, as follows:

\[
D = (CS \times TLIM) + D' \tag{1}
\]

where \(D\) is distance (in m), CS is in meters per second, TLIM is in seconds, and \(D'\) is in meters.

To introduce the inverse time model, instruct the student to plot an x-y scatterplot of speed (data in column C, y-axis) and the inverse of TLIM (data in column D, x-axis). Repeating the procedure for showing the regression trendline, the student will now discover the slope and y-intercept is reversed from those shown in the distance-time plot (Fig. 1, right). Such a configuration reveals another two-component model equation:

\[
S = [D'(1/TLIM)] + CS \tag{2}
\]

where \(S\) is speed (in m/s) and the inverse of time is \(1/TLIM\) (in s). Equation 2 also can be rearranged to remove the inverse of time portion, as follows:

\[
S = (D'/TLIM) + CS \tag{3}
\]

Finally, the student can be exposed to the mathematics for solving TLIM for a given running speed or distance. Such equations are algebraic rearrangements of Eqs. 1–3. The most succinct expression for solving TLIM is as follows:

\[
TLIM = (D - D')/CS \tag{4}
\]

However, the solution of equation also can be derived using the following equation:

\[
TLIM = 1/[(S - CS)/D'] \tag{5}
\]

At this point in the activity, it is a good idea to review the physiological factors governing CS and \(D'\). For instance, I ask “how would improving either CS or \(D'\) alter these charts?” and “what type of training regimen is optimum for improving CS or \(D'\)?”

Creating a Scatterplot of Cycling Data to Derive CP and \(W\)

With a firmer understanding of the inverse time model, have the student repeat the procedures using the cycle ergometry data shown in Table 2. It is important to point out that cadence is presumed constant (e.g., 80 rpm) because changes in rpm will affect the power output at CP. For instance, Barker et al. (1) reported lower CP values for pedaling at 100 vs. 60 rpm yet similar \(\dot{V}O_2\) values for either condition. Equations for rearranging units of the Monark cycle ergometer are also shown in Table 2. The student can then be asked to create an x-y scatterplot of power on the y-axis and the inverse of TLIM on the x-axis. Using the...
trendline feature, the power-TLIM curvature constant \( W' \) and CP are shown for the slope and \( y \)-intercept, respectively (Fig. 2, left). The units of measurement for \( W' \) and CP are in joules and watts, respectively.

In the power-TLIM plot, I am intentional in my use of three instead of four data points, as was done with Fig. 1, and I was intentional to have the regression line not fit each data point exactly. Prior scholars have argued a minimum of four bouts are needed for the optimum estimation of CP and \( W' \) \cite{8}; however, there are examples where studies using three exhaustive bouts have been published. The plot shown in Fig. 2 also helps introduce the concept that there is variability in exhaustive, constant-load exercise bouts \cite{21}. Both traditional \cite{7} and all-out estimates \cite{10} indicate that \( W' \) may fluctuate from day to day, perhaps due to glycogen storage \cite{13}. Similar test-retest variability has been observed with measures of the maximally accumulated O\(_2\) deficit \cite{4}. Conversely, CP is very reliable measurement \cite{7, 10}. Collectively, it is important to stress that the predictions rendered from the CP model, which albeit are a single value, have an inherent SE of estimate associated with them.

To illuminate the student to the hyperbolic relationship between power and TLIM, have the student create a scatterplot of power on the \( y \)-axis and TLIM on the \( x \)-axis using the time points of 150, 300, 450, and 600 s (i.e., an exponential increase in TLIM). Power will be calculated using the equation on the spreadsheet (Fig. 2, left), expressed as follows:

\[
\text{Power} = \left[ W' \left( \frac{1}{\text{TLIM}} \right) \right] + \text{CP} \tag{6}
\]

or using the following rearranged equation:

\[
\text{Power} = \left( \frac{W'}{\text{TLIM}} \right) + \text{CP} \tag{7}
\]

A second series of data plotting CP at each TLIM point is also recommended (Fig. 2, right, dashed line). I suggest using the “Insert Shape” feature in Excel to draw rectangles to represent the area (or volume) of \( W' \), a common illustration for the power-TLIM relationship \cite{11}. The geometric adjustment of these rectangles can truly illuminate student discovery. Of particular interest is the ability to rotate the rectangle at the TLIM of 150 s to a 90° position to illustrate how the rectangle extends to the TLIM of 300 s.

**Exploring Proportionality of the Severe Exercise Domain**

The severe exercise domain is defined by exercise intensities exceeding CP and of sufficient duration to enable attainment of \( \dot{V}O_2\text{max} \) \cite{3}. All-out cycling exercise has been reported to take up to 150 s to wholly deplete \( W' \) \cite{18}; therefore, 150 s was set as the minimal TLIM that I have used for this student activity. To explore the proportionality of the severe exercise domain, have the student calculate the following equation:

\[
\text{Power} = \left[ \frac{1}{2} \left( \text{Power at the TLIM of 150 s} - \text{CP} \right) \right] + \text{CP}.
\]

The student will notice that the solution is equivalent to the power at the TLIM of 300 s, as demonstrated by the rectangle shown in Fig. 2. The student can then calculate the following equation:

\[
\text{Power} = \left[ \frac{1}{3} \left( \text{Power at the TLIM of 150 s} - \text{CP} \right) \right] + \text{CP} \tag{8}
\]

The student will notice that the solution is equivalent to the power at the TLIM of 450 s. The student also can resolve the loads at each power output at CP and at these various TLIM and notice an identical proportional trend in the reduction of load relative to the load associated with CP. Specifically, with the presumption that cadence is constant, \( 1/2 \) of the difference in load doubles TLIM, \( 1/3 \) of the difference in load triples TLIM, and so forth. The equation for deriving load is as follows:

**Table 1. Sample distances and running times (TLIM) along with speed and inverse of time calculations in Excel**

<table>
<thead>
<tr>
<th>Row</th>
<th>Distance, m</th>
<th>TLIM, s</th>
<th>Speed, m/s</th>
<th>Inverse of Time, 1/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1,000</td>
<td>208.8</td>
<td></td>
<td>( \frac{A2}{B2} )</td>
</tr>
<tr>
<td>3</td>
<td>1,600</td>
<td>350.0</td>
<td></td>
<td>( \frac{A5}{B5} )</td>
</tr>
<tr>
<td>4</td>
<td>3,000</td>
<td>679.4</td>
<td></td>
<td>( \frac{A5}{B5} )</td>
</tr>
<tr>
<td>5</td>
<td>5,000</td>
<td>1,150.0</td>
<td></td>
<td>( \frac{A5}{B5} )</td>
</tr>
</tbody>
</table>

TLIM, time limit. Arrows denote the use of auto-fill option in Excel.
Load = \left[ \frac{(\text{power} \times 6.12)}{6} \right] / 80 \text{ rpm} \quad (9)

where there is 6.12 kg·min⁻¹·W⁻¹, 6 m/revolution, and 80 rpm of cadence in the example shown in Table 2.

Quantifying Proportional and Total Work or Energy

The contribution of $W'$ to overall work or energy expenditure also reduces proportionately with time. Total work or energy expenditure is calculated using the following equation:

\[
\text{Total work} = \text{power} \times \text{TLIM} \quad (10)
\]

$W'$ is divided by total work to calculate the portion derived primarily by anaerobic capacity. The aerobic proportion is derived by subtracting the anaerobic proportion from 1.0 (note: each variable can by multiplied by 100 to express results as percentages). If these proportions are calculated for the TLIMs of 150, 300, 450, and 600 s, the student will note that the reduction of $W'$ to total work is hyperbolic. Indeed, theoretically, the calculations can carried out to infinity; however, the biological limits of CP are most likely 30 min (19).

Summary

The CP model has tremendous applications from estimating performance and quantifying energy expenditure for monitoring training-induced adaptations and prescribing severe exercise bouts (11). Calculating work or energy expenditure for cycling requires the standardization of pedaling cadence and conversion factors for calculating load. Considering those iterations combined with the mathematics for the inverse time model, it comes as no surprise that the CP concept can feel like learning a new language. Evaluating plots of running distances and TLIMs compared with speed and the inverse of the TLIM serves as an effective method for introducing the parameters of CS and $D'$ along with the hyperbolic nature of the CP model.

DISCLOSURES

No conflicts of interest, financial or otherwise, are declared by the author(s).

AUTHOR CONTRIBUTIONS

R.W.P. conception and design of research; R.W.P. performed experiments; R.W.P. analyzed data; R.W.P. interpreted results of experiments; R.W.P. prepared figures; R.W.P. drafted manuscript; R.W.P. edited and revised manuscript; R.W.P. approved final version of manuscript.

REFERENCES


Table 2. Sample load and cycle ergometry times (TLIM) along with power and inverse of time calculations in Excel

<table>
<thead>
<tr>
<th>Row</th>
<th>Load, kp</th>
<th>TLIM, s</th>
<th>Power, W</th>
<th>Inverse of Time, 1/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>274</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.4</td>
<td>137</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.7</td>
<td>110</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A cadence of 80 rpm was used. Arrows denote the use of the auto-fill option in Excel.


